Approximation Algorithms for Two Optimal Location Problems in Sensor Networks

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Abstract

This paper studies two problems that arise in optimization of sensor networks: First, we devise provable approximation schemes for locating a base station and constructing a network among a set of sensors each of which has a data stream to get to the base station. Subject to power constraints at the sensors, our goal is to locate the base station and establish a network in order to maximize the lifespan of the network.

Second, we study optimal sensor placement problems for quality coverage of given domains cluttered with obstacles. Using line-of-site sensors, the goal is to minimize the number of sensors required in order to have each point "well covered" according to precise criteria (e.g., that each point is seen by two sensors that form at least angle α, or that each point is seen by three sensors that form a triangle containing the point).

1 Introduction

Consider a wireless sensor network with a large number of deployed sensors, each capturing data on a continuous basis. The sensors may be capturing video data, audio data, environmental data, etc. There is a base station that collects all of the data streams from all of the sensors. Each sensor passes data packets along some route in a network, from sensor to sensor, so that all data arrives at the base station. Since each sensor is generally powered by some form of battery, the duration of the sensor node is determined in large part by its power dissipation rate and energy provision. A fundamental issue associated with wireless sensor networks is maximizing their useful lifetime, given their power constraints. This can be significantly affected by the location of the base station as well as the forwarding protocols used (i.e. which sensor forwards packages of which other sensor) in establishing the network. Hou[1] has suggested the use of the length of time until the first sensor exhausts its battery as a definition of the *lifespan* of the system. In the paper, we show how to find a location of the base station such that the lifespan of the system is optimized to within any desired approximation bound. Specifically, we give a method for locating the base station that provably obtains a lifespan of at least $(1-\varepsilon)$ times that of the optimal lifespan, where $\varepsilon > 0$ is any pre-determined fixed value. The algorithm is based on solving $O(n\varepsilon^{-4}\log^2(n/\varepsilon))$ instances of a linear programming problem. It is simple and easy to implement.

Theorem 1.1 *Given a set of sensors $\mathcal{S} = \{s_1, \ldots, s_n\}$ and a parameter $\varepsilon > 0$, one can compute a location of the base station and a transmission scheme such that the network lifespan is at least $(1-\varepsilon)t_{opt}$, where t_{opt} is the lifespan of an optimal transmission scheme for $\mathcal{S}$. This algorithms requires $M = O(n\varepsilon^{-4}\log^2(n/\varepsilon))$ preprocessing time, and needs to solve M instances of linear programming.*

[1]Thomas Hou, personal communication, 2003.

We also study another optimal location problem in sensor networks: How does one choose locations of sensors for line-of-sight coverage of a given region, under the assumption that a point is only "covered" if it is "well seen"? We consider two definitions of "well seen": A point p is well seen (or *robustly covered*) if either (a) there are two sensors that see p, and these sensors are separated by angle at least α with respect to p; or (b) there are three sensors that see p and they form a triangle that contains p. The objective is to minimize the number of sensors to achieve robust coverage. Our results on this problem give efficient approximation algorithms for minimizing the number of sensors.

Theorem 1.2 *Given P and Q as above, an angle α, and a grid Γ of edge-length δ inside P, we can find a set G of sensors in P such that G 2-guards Q at angle $\alpha/2$, and $|G| = O(k_{opt} \log k_{opt})$, where k_{opt} is the cardinality of smallest set of vertices of Γ that 2-guard Q at angle α. The running time of the algorithm is $O(nk_{opt}^4 \log^2 n \log m)$, where m is the number of vertices of $\Gamma \cap P$.*

Theorem 1.3 *Deciding if k sensors within P suffice to triangle-guard Q is NP-hard.*

Theorem 1.4 *Given P and Q as above, and a grid Γ of edge-length δ, we can find a set G of sensors in P, that triangle-guard Q, with $|G| = O(k_{opt} \log k_{opt})$, where k_{opt} is the cardinality of smallest set of vertices of Γ that triangle-guard Q. The running time of the algorithm is $O(nk_{opt}^2 \log^2 n \log m)$, where m is the number of vertices of $\Gamma \cap P$.*

This second problem is a variant of the classical art gallery problem, in which one is to place the fewest sensors ("guards") to see all points of a certain geometric domain. Art gallery problems have been studied extensively; see, e.g., [Kei00, Urr00] for recent surveys. The algorithmic problem of computing a minimum number of guards is known to be NP-hard, even if the input domain, $\mathcal{D}$, is a simple polygon. Thus, efforts have concentrated on approximation algorithms for optimal guarding problems. Recently, researchers [EH02, GL01] have applied set cover methods that exploit finiteness of VC-dimension. In particular, Efrat and Har-Peled [EH02] obtain an $O(\log k^*)$-approximation algorithm for guarding a polygon with vertex guards, using time $O(n(k^*)^2 \log^4 n)$, where k^* is the optimal number of vertex guards. Cheong et al. [CEH04] have recently shown how to compute k guards in order to optimize (approximately) the total area seen by the guards. The triangle-guarding coverage problem we study is related to recent work of Smith and Evans [SE03].

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