# The Metric TSP and the Sum of its Marginal Values

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## Abstract

This paper examines the relation between the length of an optimal Traveling Salesman tour and the sum of its nodes' marginal values (a node's marginal value is the difference between the length of an optimal TSP tour over a given node set and the length of an optimal TSP tour over the node set minus the node). To our knowledge, this problem has not been studied previously. We find that in metric spaces $L_1, L_{4/3}, L_2, L_4, L_\infty$, the event in which the sum of TSP marginal values is greater than the length of the optimal tour is very rare. We present a number of cases for which the sum of marginal values is never greater that the optimal tour. We establish a worst case ratio of 2 for any metric TSP. In addition, for 6 node TSPs we determine the worst ratio for $L_1, L_\infty$ norms, triangular inequality, and symmetric distance, of 10/9, 10/9, 1.2, and 1.5 respectively, by solving the appropriate mixed integer programming problems.

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